NEM 2

[09-02-02-EEM1C]

(b) For $h(x) = 2$, $x^2 - 4x + 5 = 2$

$(x - 1)(x - 3) = 0$

$x = 1$ or 3

For $h(x) = 5$, $x^2 - 4x + 5 = 5$

$x(x - 4) = 0$

$x = 0$ or 4

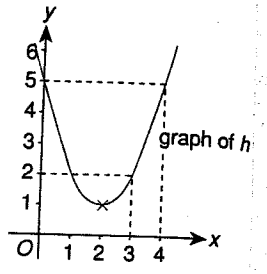


Fig. 8.11

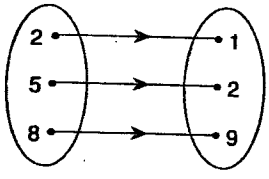
As shown in Fig. 8.11, a corresponding domain is $3 \leqslant x < 4$.

Note: Another corresponding domain is $0 < x \leqslant 1$.

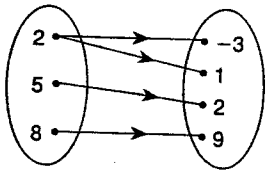
Exercise 8.4 (Answers on p. 562)

1. Which of the following relations is not a function? State your reason.

(a)



(b)

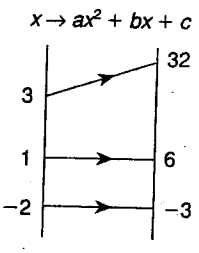


2. A function f is defined by $f: x \mapsto 4x + 2$. Find the images of -1, 3 and 5.

3. A function g is defined by $g: x \mapsto 3 + \frac{4}{x - 1}$, $x \neq 1$. Find the images of -2, $\frac{1}{2}$ and $\frac{5}{4}$.

4. A function f is defined by $f: x \mapsto ax + b$. The images of 1 and 5 are -2 and 10 respectively. Calculate the value of a and of b.

5. The arrow diagram shows part of the function $f: x \mapsto ax^2 + bx + c$.

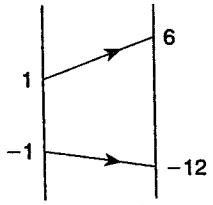


Find

(a) the values of a, b and c,

(b) the positive number x whose image is 2.

6. The arrow diagram shows part of the mapping $f: x \mapsto \frac{24}{ax+b}$, $x \neq -\frac{b}{a}$, $a \neq 0$.



(a) Find the value of a and of b.
(b) Find the element that has an image of 8 under this mapping.
(c) Find the two values of x for which $f(x) = x$.

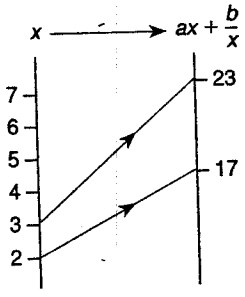
7. Given the function $f: x \mapsto \frac{2}{ax+b}$, $x \neq \frac{-b}{a}$ such that $f(0) = -2$ and $f(2) = 2$, find

(a) the value of a and of b,
(b) the values of x for which $f(x) = x$.

Show that $f(p) + f(-p) = 2f(p^2)$.

8. The arrow diagram represents part of the mapping $x \mapsto ax + \frac{b}{x}$, $x \neq 0$. Find

(a) the value of a and of b,
(b) the image of 1 under this mapping.



9. The functions f are defined as follows:

(a) $f: x \mapsto 5x - 1$,
(b) $f: x \mapsto 2x + 1$,
(c) $f: x \mapsto 5 - x$,
(d) $f: x \mapsto 7 - 2x$.

For each of the functions, find

(i) the range corresponding to a domain of $1 \leqslant x \leqslant 3$,
(ii) the domain corresponding to a range of $-1 \leqslant f(x) \leqslant 5$.

10. The function f is defined for real values of x by $f: x \mapsto x^2 - 2x$.

(a) Find the range of f corresponding to the domain $-2 \leqslant x \leqslant 0$.
(b) State another domain for which f has the same range.

11. (a) Given that $f: x \mapsto 4x^2 - 3$, find a domain of x corresponding to the range $1 < f(x) < 13$.

(b) Given that $g: x \mapsto (x+1)^2$, find the range of g corresponding to the domain

(i) $0 \leqslant x \leqslant 2$, (ii) $-2 \leqslant x \leqslant 2$.

12. A function f is defined by $f: x \mapsto 2(x-1)^2 + 1$.

(a) Find the range corresponding to the domain $0 \leqslant x \leqslant 3$.

(b) Find a domain of x corresponding to the range $3 \leqslant f(x) \leqslant 5$.

13. Two functions f and g are defined by $f: x \mapsto \frac{1}{x+1}$, $x \neq -1$, and $g: x \mapsto \frac{x}{x-2}$, $x \neq 2$.

Find the values of x for which $f(x) = 10\,g(x)$.

14. Two functions f and g are defined by $f: x \mapsto \frac{3}{ax-b}$, $x \neq \frac{b}{a}$, and $g: x \mapsto bx + a$, where a and b are positive constants. Given that $f(1) = g(1)$ and $f(2)g(2) = 4$, find the value of a and of b.

15. Two functions are defined by $f: x \mapsto ax + 1$ and $g: x \mapsto \frac{4b}{x-1}$, $x \neq 1$, where a and b are constants. Given that $f(a) = g(b)$ and $f\left(\frac{1}{a}\right) = g\left(\frac{1}{b}\right)$, find the possible values of a and b.

$$
\begin{aligned}
f^3(x) &= ff^2(x) \\
&= f(f^2(x)) \\
&= f\left(\frac{4x}{x+1}\right), \quad x \neq 1, -1 \\
&= \frac{2\left(\frac{4x}{x+1}\right)}{\left(\frac{4x}{x+1}\right) - 1} \\
&= \frac{\frac{8x}{x+1}}{\frac{4x-(x+1)}{x+1}} \\
&= \underline{\underline{\frac{8x}{3x-1}, \quad x \neq \frac{1}{3}}}
\end{aligned}
$$

(b) f^2 is not defined for $\underline{\underline{x = 1 \text{ and } x = -1}}$.

f^3 is not defined for $\underline{\underline{x = 1, x = -1 \text{ and } x = \frac{1}{3}}}$.

Exercise 8.2 *(Answers on p. 563)*

1. For each of the following pairs of functions, obtain expressions in the same form for gf and fg.

(a) $f: x \mapsto 3x, \quad g: x \mapsto 3 - 2x$

(b) $f: x \mapsto 2x + 1, \quad g: x \mapsto 2 - x^2$

(c) $f: x \mapsto x - 4, \quad g: x \mapsto \frac{2}{x}, \quad x \neq 0$

(d) $f: x \mapsto 1 + 2x, \quad g: x \mapsto \frac{x}{x-1}, \quad x \neq 1$

2. For each of the following functions, obtain expressions in the same form for f^2 and f^3.

(a) $f: x \mapsto 2x + 3$

(b) $f: x \mapsto \frac{3x}{x-1}, \quad x \neq 1$

(c) $f: x \mapsto \frac{x}{x-1}, \quad x \neq 1$

(d) $f: x \mapsto \frac{3}{2x-1}, \quad x \neq \frac{1}{2}$

3. Functions f and g are defined by $f: x \mapsto 3x + 4, \quad g: x \mapsto x^2 + 6$.
Using this notation, obtain expressions for fg and gf. Find the values of x for which

(a) $f = g$,

(b) $fg = gf$.

4. Two functions f and g are defined by $f: x \mapsto 2x + p$, $g: x \mapsto 3x + q$.
If $fg = gf$ for all values of x, find the relation between p and q. Given that $p = 3$, find the values of x for which
(a) $f^2 = g$, **(b)** $g^2 = f$.

5. Two functions are defined by $f: x \mapsto 2x - 1$, $g: x \mapsto \frac{x}{x+1}$, $x \neq -1$.
Obtain expressions in the same form for fg and gf. Find the value of x for which $fg = gf$.

6. A function f is defined by $f: x \mapsto x + 1$. Another function g is such that $gf: x \mapsto x^2 + 2x + 5$. Express $gf(x)$ in the form $a(x + 1)^2 + b$ and hence write down the expression for $g(x)$.

7. Two functions are defined by $f: x \mapsto \frac{1}{x+1}$, $x \neq -1$, $x \neq -\frac{1}{2}$, and $g: x \mapsto \frac{x}{x-2}$, $x \neq 2$, $x \neq 1$. Obtain expressions in similar form for fg and gf. Find the values of x for which $fg(x) + gf(x) = 0$.

***8.** A function f is defined by $f: x \mapsto \frac{x}{x+1}$, $x \neq -1$. Prove that $f^2(x) = \frac{x}{2x+1}$, $x \neq -1$, $x \neq -\frac{1}{2}$, and obtain a similar expression for $f^3(x)$. Hence suggest a possible expression for $f^n(x)$.

9. The function f is defined by $f: x \mapsto \frac{x+1}{x-1}$, $x \neq 1$. Show that $f^2(x) = x$, $x \neq 1$, and hence express $f^5(x)$ and $f^{10}(x)$ in their simplest forms.

10. Two functions are defined by $f: x \mapsto x - 2$ and $g: x \mapsto \frac{2}{x}$, $x \neq 0$. Express in similar form,
(a) f^2, **(b)** f^5, **(c)** g^2, **(d)** g^5.

11. A function $f: x \mapsto \frac{b}{x-a}$, $a > 0$, is such that $f(b) = b$ and $f(2a) = 2a$. Find the value of a and of b. If $ff(x) = x$, show that $x^2 - x - 2 = 0$.

12. A function f is defined by $f: x \mapsto \frac{2}{x} + k$, $x \neq 0$, where k is a constant. Given that $f^2(2) = \frac{2}{3}f(1)$, calculate the possible values of k.

Observe that the point $B(3, 1)$ on the graph of f^{-1} is the reflection of $A(1, 3)$ on the graph of f in the line $y = x$.

In general, any point (b, a) on the graph of f^{-1} is the reflection of the point (a, b) on the graph of f in the line $y = x$. Geometrically, the graph of f^{-1} is the reflection of the graph of f in the line $y = x$.

Example 13 A function f is defined by $f: x \mapsto 3 - 2x$.
Sketch the graph of f for the domain $-1 \leqslant x \leqslant 3$ and hence sketch the corresponding graph of f^{-1}.

Solution:

$$f(x) = 3 - 2x$$
$$f(-1) = 3 - 2(-1) = 5$$
$$f(3) = 3 - 2(3) = -3$$

The graph of f is a line segment with end-points $(-1, 5)$ and $(3, -3)$. Reflecting the graph of f in the line $y = x$, we obtain the graph of f^{-1} which is a line segment with end-points $(5, -1)$ and $(-3, 3)$ as shown in Fig. 8.20.

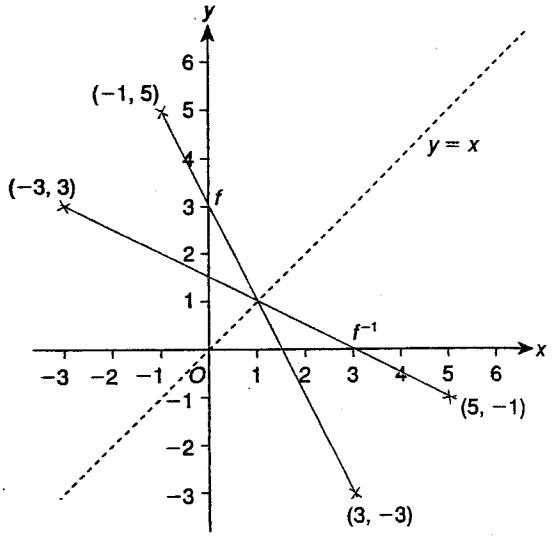
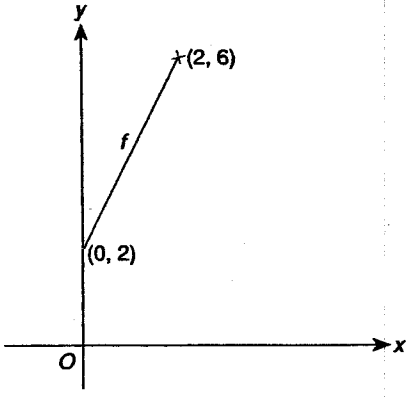


Fig. 8.20

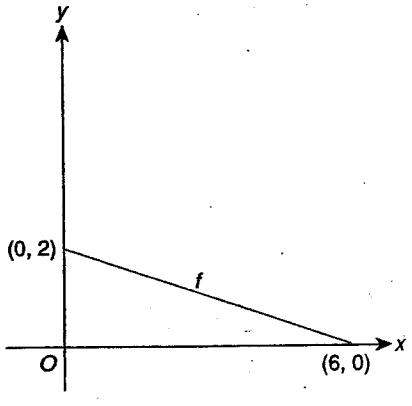
Exercise 8.3 (Answers on p. 563)

1. Each of the following diagrams shows the graph of a function f. In each case, sketch the graph of f^{-1}.

(a)



(b)



2. Find f^{-1} in similar form for each of the following functions.

(a) $f: x \mapsto 3x - 2$

(b) $f: x \mapsto \frac{3}{x-1}, x \neq 1$

(c) $f: x \mapsto \frac{2x}{x-2}, x \neq 2$

(d) $f: x \mapsto \frac{2x+3}{2x-1}, x \neq \frac{1}{2}$

3. A function f is defined by $f: x \mapsto \frac{4x-9}{x-2}, \quad x \neq 2.$

(a) Find $f^{-1}(-1)$ and $f^{-1}(1)$.
(b) Find the value of x for which $4f^{-1}(x) = x$.

4. A function f is defined by $f: x \mapsto \frac{2x+2}{x-1}, \quad x \neq 1.$

(a) Find $f^{-1}(3)$.
(b) Given that $f^{-1}(p) = kp$, express k in terms of p.

5. Given the function $f: x \mapsto 6x - \frac{2}{x}, \quad x > 0$, find the value of

(a) $f^{-1}(-1)$,

(b) $f^{-1}(-4)$.

For what value of x is $f(x) = 1$?

6. For the function defined by $f: x \mapsto \frac{a}{x-1} + b$, state the value of x for which f is not defined. Given that $f(2) = 3$ and $f(3) = 2$,

(a) find the value of a and of b,
(b) show that $ff(x) = x$,
(c) find $f^{-1}(x)$.

7. The functions f and g are defined by $f: x \mapsto 2x + 3, \quad g: x \mapsto \frac{1}{x}, \quad x \neq 0.$

Write down in similar form, expressions for fg, gf, f^{-1}, g^{-1}. Find the value of x for which $fg(x) = g^{-1}(x)$.

8. A function is defined by $f: x \mapsto \frac{2x+1}{x-1}$ for all values of x except $x = 1$. Express the function f^{-1} in similar form and state the value of x for which f^{-1} is not defined. Find the values of

(a) $f^{-1}(3)$,

(b) x for which $f(x) = 4f^{-1}(x)$.

9. Functions f and g are defined on the set of real numbers by $f: x \mapsto \frac{3}{x+2}, x \neq k$, and $g: x \mapsto 2x + 1$.

(a) State the value of k.
(b) Express fg in similar form and state the value of x for which fg is not defined.
(c) Find the value of p for which $f^{-1}(p) = g(9)$.

10. A function f is defined by $f: x \mapsto \frac{3x}{x-3}$ for all values of x except $x = 3$. Show that $ff(x) = x$ for all values of x except $x = 3$. Find $f(5)$ and $f^{-1}(5)$.

11. A function f is defined by $f: x \mapsto \frac{x+1}{x-1}$, $x \neq 1$. If $f^2(2) = 3f^{-1}(a)$, find the value of a.

12. A function f is defined by $f: x \mapsto \frac{a}{x-1}$, $x \neq 1$. If $f(a) + f^{-1}(2a) = 1$, find the value of a.

13. Functions f and g are defined by $f: x \mapsto \frac{2x}{x-1}$, $x \neq 1$, and $g: x \mapsto \frac{x+a}{x}$, $x \neq 0$. Find f^{-1} in similar form. Given that $gf^{-1}(3) = 4$, calculate the value of a.

Refer to Fig. 8.21, $(gf)^{-1} : z \mapsto x$ and $f^{-1}g^{-1} : z \mapsto x$, we have

$(gf)^{-1} = f^{-1}g^{-1}$ holds for any two functions.

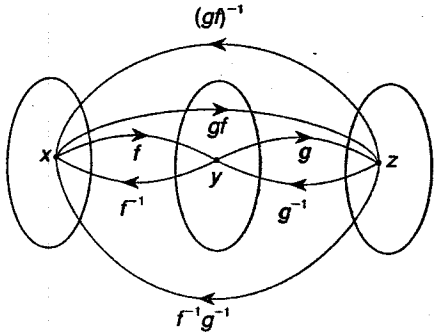


Fig. 8.21

Exercise 8.4 (Answers on p. 564)

1. Given the functions $f : x \mapsto 2x + 1$ and $g : x \mapsto 5 - x$, express the following functions in similar form.

(a) ff **(b)** g^{-1} **(c)** fg

(d) $(fg)^{-1}$ **(e)** f^{-1} **(f)** $g^{-1}f^{-1}$

2. A function f is defined by $f : x \mapsto \frac{3}{x}$, $x \neq 0$.

Obtain expressions for the following in similar form.

(a) f^{-1} **(b)** f^2 **(c)** $(f^2)^{-1}$ **(d)** $(f^{-1})^2$

3. A function is defined by $f : x \mapsto \frac{1}{2x + 1}$, $x \neq -\frac{1}{2}, -\frac{3}{2}$.

(a) Write in similar form expressions for f^2 and f^{-1}.

(b) Show that $(f^2)^{-1}(x) = (f^{-1})^2(x)$.

4. The functions f and g are defined by $f : x \mapsto 2x + 3$ and $g : x \mapsto 3x - 2$. Write down, in similar form, fg, f^{-1}, g^{-1}, and show that $g^{-1}f^{-1} = (fg)^{-1}$.

5. Two functions are defined by $f : x \mapsto 3x + 1$ and $g : x \mapsto \frac{2}{x}$ (for $x \neq 0$). Find in similar form

(a) fg and $(fg)^{-1}$, **(b)** f^{-1}, g^{-1} and $g^{-1}f^{-1}$.

6. Given the functions $f : x \mapsto 2x - 3$ and $g : x \mapsto \frac{1}{x - 1}$ (for $x \neq 1$), find in similar form

(a) fg and $(fg)^{-1}$, **(b)** f^{-1}, g^{-1} and $g^{-1}f^{-1}$.

Is $(fg)^{-1} = g^{-1}f^{-1}$?

(f) Plot lg y against lg x, b = gradient, $-\lg a$ = (lg y)-intercept
(g) Plot lg y against x, $-\lg q$ = gradient, $\lg p$ = (lg y)-intercept
(h) Plot $\frac{e^y}{x}$ against x; p = gradient, $-q = \left(\frac{e^y}{x}\right)$-intercept

8. Plot 10^y against x, a = gradient, $b = 10^y$-intercept
9. $h = 10$, $k = -2$ **10.** $a \approx 1.58$, $b \approx 2.67$ **11.** $a \approx 2$, $b \approx -0.4$
12. $C \approx 1.2$, $D \approx 1.5$ **13.** Plot lg y against x, $k \approx 12$, $b \approx 2.5$
14. $\lg y = b \lg x + \lg a$; $a \approx 0.3$, $b \approx 2.5$

Miscellaneous Exercise 7 *(p. 118)*

1. (a) $y = \frac{15}{2x+3}$ (b) $y = 1\,000x^2$ **2.** $h = 4$, $k = -16$, $r = 4$
3. $a = 3$, $b = -4$ **4.** Plot y^2 against x, a = gradient, $b = y^2$-intercept
5. (a) 4 (b) $y = \frac{3x+8}{2x^2}$ **6.** (a) $p \approx -3$, $q \approx 1.5$ (b) $x \approx 3.4$
7. $xy = ax^2 + b$ (a) $a \approx -0.67$, $b \approx 7.47$ (b) $y \approx 3.26$
8. (a) 25.1; 17.78 (b) $C \approx 100$, $a \approx 1.78$ (c) $x \approx 8$
9. (a) Plot y^2 against x^2. If the graph is a straight line, then the equation is valid and $-\frac{a}{b}$ = gradient, $\frac{1}{b} = y^2$-intercept.
(b) Plot lg y against lg x. If the graph is a straight line, then the equation is valid and d = gradient, lg c = lg y-intercept.
10. $A \approx 21$, $b \approx 0.56$ **11.** (a) $a \approx 2$, $b \approx 1.3$ (b) ± 0.92 (c) 0.28
12. $k = 7.39$, $p = 0.472$ **13.** (a) $a \approx 2$, $b \approx 0.5$ (b) 1.90 s (c) 0.25 m
14. (a) 0.91, 1.00 (b) 0.5 (c) $p \approx -1$, $q \approx 0.5$
15. (a) 2.83 (b) $a \approx -0.4$, $b \approx -2$
16. (a) $m \approx 3$, $n \approx 2$ (b) 1.26

CHAPTER 8

Exercise 8.1 *(p. 128)*

1. The relation in (b) is not a function as $2 \mapsto -3$ and $2 \mapsto 1$.
2. -2, 14, 22 **3.** $\frac{5}{3}$, -5, 19 **4.** $a = 3$, $b = -5$
5. (a) $a = 2$, $b = 5$, $c = -1$ (b) $\frac{1}{2}$
6. (a) $a = 3$, $b = 1$ (b) $\frac{2}{3}$ (c) $2\frac{2}{3}$, -3
7. (a) $a = 1$, $b = -1$ (b) -1, 2 **8.** (a) $a = 7$, $b = 6$ (b) 13
9. (a) (i) $4 \leqslant f(x) \leqslant 14$ (ii) $0 \leqslant x \leqslant \frac{6}{5}$ (b) (i) $3 \leqslant f(x) \leqslant 7$ (ii) $-1 \leqslant x \leqslant 2$
(c) (i) $2 \leqslant f(x) \leqslant 4$ (ii) $0 \leqslant x \leqslant 6$ (d) (i) $1 \leqslant f(x) \leqslant 5$ (ii) $1 \leqslant x \leqslant 4$
10. (a) $0 \leqslant f(x) \leqslant 8$ (b) $2 \leqslant x \leqslant 4$
11. (a) $1 < x < 2$ (b) (i) $1 \leqslant g(x) \leqslant 9$ (ii) $0 \leqslant g(x) \leqslant 9$
12. (a) $1 \leqslant f(x) \leqslant 9$ (b) $2 \leqslant x \leqslant \sqrt{2} + 1$ or $1 - \sqrt{2} \leqslant x \leqslant 0$
13. $-\frac{1}{2}$, $-\frac{2}{5}$ **14.** $a = 2$, $b = 1$ **15.** $a = 1$, $b = -1$; $a = -1$, $b = -1$.

Exercise 8.2 *(p. 133)*

1. (a) $gf: x \mapsto 3 - 6x$; $fg: x \mapsto 9 - 6x$ (b) $gf: x \mapsto 1 - 4x - 4x^2$; $fg: x \mapsto 5 - 2x^2$

(c) $gf: x \mapsto \frac{2}{x-4}$, $x \neq 4$; $fg: x \mapsto \frac{2}{x} - 4$, $x \neq 0$

(d) $gf: x \mapsto \frac{1+2x}{2x}$, $x \neq 0$; $fg: x \mapsto \frac{3x-1}{x-1}$, $x \neq 1$

2. (a) $f^2: x \mapsto 4x + 9$; $f^3: x \mapsto 8x + 21$

(b) $f^2: x \mapsto \frac{9x}{2x+1}$; $x \neq 1, -\frac{1}{2}$; $f^3: x \mapsto \frac{27x}{7x-1}$, $x \neq 1, -\frac{1}{2}, \frac{1}{7}$

(c) $f^2: x \mapsto x$, $x \neq 1$; $f^3: x \mapsto \frac{x}{x-1}$, $x \neq 1$

(d) $f^2: x \mapsto \frac{3(2x-1)}{7-2x}$, $x \neq \frac{1}{2}, \frac{7}{2}$; $f^3: x \mapsto \frac{3(7-2x)}{14x-13}$, $x \neq \frac{1}{2}, \frac{7}{2}, \frac{13}{14}$

3. $fg: x \mapsto 3x^2 + 22$; $gf: x \mapsto 9x^2 + 24x + 22$ (a) 1, 2 (b) 0, −4

4. $q = 2p$ (a) −3 (b) −3

5. $fg: x \mapsto \frac{x-1}{x+1}$, $x \neq -1$; $gf: x \mapsto \frac{2x-1}{2x}$, $x \neq 0$; $x = \frac{1}{3}$ **6.** $g(x) = x^2 + 4$

7. $fg: x \mapsto \frac{x-2}{2(x-1)}$; $x \neq 1, 2$; $gf: x \mapsto -\frac{1}{1+2x}$, $x \neq -1, -\frac{1}{2}$ (b) $0, \frac{5}{2}$

8. $f^3(x) = \frac{x}{3x+1}$, $x \neq -1$, $x \neq -\frac{1}{2}$, $x \neq -\frac{1}{3}$; $f^n(x) = \frac{x}{nx+1}$, $x \neq -1$, $x \neq -\frac{1}{2}$, $x \neq -\frac{1}{3}$, $x \neq -\frac{1}{4}$, ... $x \neq -\frac{1}{(n-1)}$, $x \neq -\frac{1}{n}$

9. $f^5(x) = \frac{x+1}{x-1}$, $x \neq 1$; $f^{10}(x) = x$, $x \neq 1$

10. (a) $f^2: x \mapsto x - 4$ (b) $f^5: x \mapsto x - 10$ (c) $g^2: x \mapsto x$ (d) $g^5: x \mapsto \frac{2}{x}$

11. $a = 1$, $b = 2$ **12.** 1, 2

Exercise 8.3 *(p. 138)*

2. (a) $f^{-1}: x \mapsto \frac{1}{3}(x + 2)$ (b) $f^{-1}: x \mapsto \frac{3+x}{x}$, $x \neq 0$

(c) $f^{-1}: x \mapsto \frac{2x}{x-2}$, $x \neq 2$ (d) $f^{-1}: x \mapsto \frac{x+3}{2(x-1)}$, $x \neq 1$

3. (a) $\frac{11}{5}, \frac{7}{3}$ (b) 6 **4.** (a) 5 (b) $\frac{p+2}{p(p-2)}$

5. (a) $\frac{1}{2}$ (b) $\frac{1}{3}, \frac{2}{3}$ **6.** (a) $a = 2$, $b = 1$ (c) $f^{-1}(x) = \frac{x+1}{x-1}$, $x \neq 1$

7. $fg: x \mapsto \frac{2}{x} + 3$, $x \neq 0$; $gf: x \mapsto \frac{1}{2x+3}$, $x \neq -\frac{3}{2}$; $f^{-1}: x \mapsto \frac{x-3}{2}$; $g^{-1}: x \mapsto \frac{1}{x}$, $x \neq 0$; $x = -\frac{1}{3}$

8. $f^{-1}: x \mapsto \frac{x+1}{x-2}$, $x \neq 2$ (a) 4 (b) $\frac{1}{2}, -2$

9. (a) −2 (b) $fg: x \mapsto \frac{3}{2x+3}$, $x \neq -\frac{3}{2}$ (c) $\frac{1}{7}$

10. $f(5) = 7.5$, $f^{-1}(5) = 7.5$ **11.** −5 **12.** $\frac{1}{3}$

13. $f^{-1}: x \mapsto \frac{x}{x-2}$; $x \neq 2$; $a = 9$

Exercise 8.4 *(p.142)*

1. (a) $ff: x \mapsto 4x + 3$ (b) $g^{-1}: x \mapsto 5 - x$ (c) $fg: x \mapsto 11 - 2x$

(d) $(fg)^{-1}: x \mapsto \frac{1}{2}(11 - x)$ (e) $f^{-1}: x \mapsto \frac{1}{2}(x - 1)$ (f) $g^{-1}f^{-1}: x \mapsto \frac{1}{2}(11 - x)$

2. (a) $f^{-1}: x \mapsto \frac{3}{x}$, $x \neq 0$ (b) $f^2: x \mapsto x$, $x \neq 0$ (c) $(f^2)^{-1}: x \mapsto x$, $x \neq 0$

(d) $(f^{-1})^2: x \mapsto x$, $x \neq 0$

3. (a) $f^2: x \mapsto \frac{2x+1}{2x+3}$, $x \neq -\frac{1}{2}, -\frac{3}{2}$, $f^{-1}: x \mapsto \frac{1-x}{2x}$, $x \neq 0$, $x \neq -\frac{1}{2}$

4. $fg: x \mapsto 6x - 1$; $f^{-1}: x \mapsto \frac{x-3}{2}$; $g^{-1}: x \mapsto \frac{x+2}{3}$

5. (a) $fg: x \mapsto \frac{6}{x} + 1$, $x \neq 0$; $(fg)^{-1}: x \mapsto \frac{6}{x-1}$, $x \neq 1$

(b) $f^{-1}: x \mapsto \frac{x-1}{3}$; $g^{-1}: x \mapsto \frac{2}{x}$, $x \neq 0$; $g^{-1}f^{-1}: x \mapsto \frac{6}{x-1}$, $x \neq 1$

6. (a) $fg: x \mapsto \frac{5-3x}{x-1}$, $x \neq 1$; $(fg)^{-1}: x \mapsto \frac{5+x}{x+3}$, $x \neq -3$, $x \neq 1$

(b) $f^{-1}: x \mapsto \frac{x+3}{2}$; $g^{-1}: x \mapsto \frac{1}{x} + 1$, $x \neq 0$; $g^{-1}f^{-1}: x \mapsto \frac{x+5}{x+3}$, $x \neq -3$; Yes

7. $f^{-1}: x \mapsto \frac{5-x}{2}$; $g^{-1}: x \mapsto \frac{1+x}{1-x}$, $x \neq 1$; $g^{-1}f^{-1}: x \mapsto \frac{7-x}{x-3}$, $x \neq 3$;

$(fg)^{-1}: x \mapsto \frac{7-x}{x-3}$, $x \neq 3$

Exercise 8.5 *(p.146)*

1. 7, 5; $-\frac{2}{3}, \frac{4}{3}$ **2.** 1, 3 **4.** $0 \leqslant f(x) \leqslant 3$; 1

5. $1 \leqslant f(x) < 8$; $\frac{3}{2}$, 0 **6.** $\frac{1}{3} \leqslant x \leqslant 3$ **7.** $x < -2$ or $x > 4$

8. $y \geqslant 1, \frac{9}{2}$ **9.** $-4 \leqslant y \leqslant 0$ **10.** $4 \leqslant x \leqslant 5$

11. 0, 2, 4 **12.** $a = 1$, $b = 1$

Miscellaneous Exercise 8 *(p. 151)*

1. (a) $p = 3$, $q = -1$ (b) 8 (c) 3 **2.** $a = 3$; $x = -1, 3$

3. (a) $f^{-1}: x \mapsto 2 - x$ (b) $ff: x \mapsto x$ (c) $gg: x \mapsto x$, $x \neq 0$

(d) $fg: x \mapsto \frac{2x-3}{x}$, $x \neq 0$ (e) $gfg: x \mapsto \frac{3x}{2x-3}$, $x \neq 0, \frac{3}{2}$

4. (a) $ff: x \mapsto 16x - 15$ (b) $gf: x \mapsto \frac{8x-11}{4x-3}$, $x \neq \frac{3}{4}$

(c) $f^{-1}: x \mapsto \frac{1}{4}(x + 3)$ (d) $g^{-1}: x \mapsto \frac{5}{2-x}$, $x \neq 2$

(e) $(fg)^{-1}: x \mapsto \frac{20}{5-x}$, $x \neq 5$

5. (a) 7 (b) -5 (c) 2 (d) $g^{-1}: x \mapsto \frac{3-x}{x+1}$, $x \neq -1$

(e) $fg: x \mapsto -\frac{2x}{x+1}$, $x \neq -1$ (f) $gfg: x \mapsto \frac{5x+3}{1-x}$, $x \neq -1, 1$

6. $f^3: x \mapsto \frac{x}{1-3x}$, $x \neq 1, \frac{1}{2}, \frac{1}{3}$ **7.** (b) $q = 5$ **8.** $p = 2$, $q = -1$.